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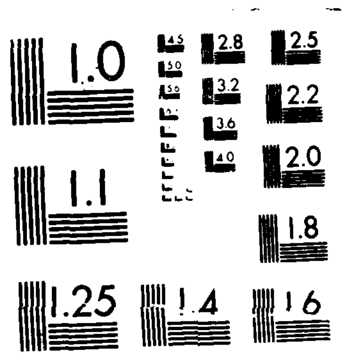
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Naval Research Laboratory

Washington, DC 20375-5000

NRL Memorandum Report 5819

AD-A180 035

Effects of Radiation Damping on Beam Quality in the Inverse Free Electron Laser Accelerator

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April 7, 1987

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AD-A180035

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S) NRL Memorandum Report 5819	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION Naval Research Laboratory 6b OFFICE SYMBOL (If applicable) Code 4790	7a NAME OF MONITORING ORGANIZATION
6c ADDRESS (City, State, and ZIP Code) Washington, DC 20375-5000	7b ADDRESS (City, State, and ZIP Code)
8a NAME OF FUNDING/SPONSORING ORGANIZATION Department of Energy 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c ADDRESS (City, State, and ZIP Code) Washington, DC 20545	10 SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO.: PROJECT NO.: DE-AI05-83ER40117 TASK NO.: WORK UNIT ACCESSION NO.: DN380-537

11 TITLE (Include Security Classification)
Effects of Radiation Damping on Beam Quality in the Inverse Free Electron Laser Accelerator

12 PERSONAL AUTHOR(S)
Ting,* A.C. and Sprangle, P.A.

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Interim	FROM ______ TO ______	1987 April 7	29

16 SUPPLEMENTARY NOTATION
*Berkeley Scholars, Inc., P.O. Box 852, Springfield, VA 22150

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Beam Quality
			Radiation Damping
			IFEL Accelerator

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

The effect of synchrotron radiation damping on the transverse beam emittance for an inverse free electron laser (IFEL) accelerator is studied. A beam envelope equation is derived and solved for an arbitrarily tapered wiggler field. The expression for the evolution of the normalized transverse beam emittance is derived and found to decrease exponentially with distance due to radiation damping, until it is limited by quantum excitation. The beam envelope equation is integrated both analytically and numerically for a set of IFEL accelerator parameters. Our results show that for acceleration distances comparable to the radiation damping e-folding length, substantial improvements in the beam quality can be realized.

20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a NAME OF RESPONSIBLE INDIVIDUAL Phillip A. Sprangle	22b TELEPHONE (Include Area Code) 202-767-3493 22c OFFICE SYMBOL Code 4790

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

CONTENTS

EFFECTS OF RADIATION DAMPING ON BEAM QUALITY IN THE INVERSE FREE ELECTRON LASER ACCELERATOR

Introduction

Electron beam quality as measured by the transverse emittance is usually determined by the gun and propagation configurations in accelerators. Under idealized conditions, the transverse normalized beam emittance remains a constant of motion as the beam propagates through the accelerator. Therefore, to improve the quality of the beam, it is necessary to decrease the beam emittance at the injection point. However, since the normalized beam emittance is essentially the transverse area in phase space for the collection of beam particles, one can in principle reduce the emittance if a dissipative mechanism is introduced. A natural candidate for such a dissipation mechanism is the induced synchrotron radiation damping due to the transverse motion of the particles in an external periodic transverse magnetic field. It is this mechanism that we will focus on when the external magnetic field is chosen to be a helical wiggler field. Since this radiation damping effect is small at low energies, it is in the context of the recently proposed high energy IFEL accelerators[1-11] that we will concentrate in this paper.

We begin by obtaining the electron orbits in an IFEL accelerator. A fully relativistic formulation of the equations of motion which include radiation damping force is considered. The damping coefficients are obtained from the transverse dynamics of the particles while the axial dynamics describes the acceleration of the particles. In the second section, a

Manuscript approved May 9, 1986

relativistic envelope equation for the average radius of the electron beam is derived, assuming continuous emission of the synchrotron radiation. It is apparent from this envelope equation that the normalized transverse emittance decays exponentially at a rate given by the radiation damping coefficient. The envelope equation is solved using a WKB method in the third section and the spatial evolution of the beam radius is obtained. Quantum excitation sets a minimum value on the normalized transverse emittance in an IFEL accelerator and it is derived in the fourth section. Strong focusing is found to be necessary to reduce such minimum to an acceptable value. An example is given in the last section for a set of proposed IFEL accelerator parameters.[2] It is found that radiation damping does reduce the emittance of the accelerated electron beam while resulting in an insignificant loss in particle energy.

I. Single Particle Dynamics

We shall consider the motion of an electron under the influence of a helical wiggler field and a circularly polarized electromagnetic wave with the inclusion of the radiation reaction force. The fully relativistic equation of motion is [14]

$$\frac{d\underline{p}}{dt} = -|e|(\underline{E} + \frac{\underline{v} \times \underline{B}}{c}) + \underline{F}^R , \qquad (1)$$

where

$$\underline{F}^R = \tau_R\{\frac{\underline{p}}{m_o^2c^2}[(\frac{\gamma^2-1}{\gamma})(\frac{d|\underline{p}|}{dt})^2 - \gamma(\frac{d\underline{p}}{dt})^2] + \frac{d}{dt}(\gamma\frac{d\underline{p}}{dt})\} ,$$

is the radiation damping force, $\tau_R = 2|e|^2/3m_oc^3$, and $\gamma^2 = 1 + |\underline{p}|^2/m_o^2c^2$. The radiation field is given by its vector potential $\underline{A}_L = A_L(\cos\phi\hat{e}_x - \sin\phi\hat{e}_y)$, where $\phi = kz-\omega t$. We shall assume z-dependence for both the magnitude and period of the wiggler field. The vector potential $\underline{A}_w$ for the helical wiggler field is given by $\underline{A}_w = A_w[\cos\theta\hat{e}_x + \sin\theta\hat{e}_y]$ where $A_w = A_w(z)$ and $\theta = \int_0^z k_w(z')dz'$.

The requirement that the wiggler field satisfies both $\nabla \cdot \underline{B}_w = 0$ and $\nabla \times \underline{B}_w = 0$ introduces transverse variation as well as a nonzero z-component of the magnetic field.[15]

Since we shall be primarily interested in laser driven accelerators, the normalized wiggler field strength $a_w = |e|A_w/m_o c^2$ is assumed to be much greater than the corresponding quantity $a_L = |e|A_L/m_o c^2$ for the radiation, i.e., $a_w >> a_L$. It can then be shown that the major contribution to the radiation damping is from the wiggler field.

We shall first look at the radiation damping term in Eq. (1). By neglecting the transverse dependence of the wiggler field for a beam that is confined sufficiently close to the axis, we have the immediate consequence that the canonical momenta in the x and y directions are constants of motion and may be chosen to be zero. The mechanical momenta are then given by $p_x = \frac{|e|}{c} \underline{A}_T \cdot \hat{e}_x$, $p_y = \frac{|e|}{c} \underline{A}_T \cdot \hat{e}_y$, where $\underline{A}_T = \underline{A}_w + \underline{A}_L$. Also, in the zeroth order approximation, the total relativistic energy is conserved which leads to $\dot{\gamma} = 0$ and $\dot{p}_z = 0$. Therefore, the only significant term remaining in the radiation reaction force is

$$\underline{F}^R \approx \tau_R \gamma [\frac{d^2 \underline{p}}{dt^2} - \frac{\underline{p}}{m_o^2 c^2} (\frac{d\underline{p}}{dt})^2].$$

Neglecting terms of order $a_L/a_w << 1$, the components of the radiation reaction force are $F_x^R = - \nu_\perp c p_x$, $F_y^R = - \nu_\perp c p_y$, $F_z^R = - \nu_\parallel c p_z$, where

$$\nu_\perp = \tau_R \gamma k_w^2 c (a_w^2 + 1), \tag{2a}$$

$$\nu_\parallel = \tau_R \gamma k_w^2 c a_w^2 , \tag{2b}$$

are respectively the spatial decay coefficients due to radiation damping in the transverse and axial directions. Note that $\nu_\perp \approx \nu_\parallel$ for $a_w^2 \gg 1$ which is the case in the IFEL accelerator.

The most significant feature of the transverse motions of the electrons is the betatron oscillation caused by either the inhomogeneity of the wiggler field in the transverse plane or other focusing mechanisms. It can be shown that, for small oscillations about the axis of the wiggler field, the transverse equations of motion are,

$$\frac{d^2x}{dz^2} + K_B^2 x = -\left(\frac{\gamma'}{\gamma} + \nu_\perp\right)\frac{dx}{dz}\,, \tag{3a}$$

$$\frac{d^2y}{dz^2} + K_B^2 y = -\left(\frac{\gamma'}{\gamma} + \nu_\perp\right)\frac{dy}{dz}\,, \tag{3b}$$

where $d/dt \approx v_z \partial/\partial z$, $v_z \approx c$, $' \equiv \partial/\partial z$ have been used, and K_B is the wave number of the longitudinal betatron oscillation. For betatron oscillations that are originated from the $\underline{v} \times \underline{B}$ force due to the nonzero magnetic field in the z-direction of the realizable wiggler field,[15] $K_B = a_w k_w/(\sqrt{2}\gamma)$.

The axial motion of the electron is governed by

$$\frac{dp_z}{dz} = \gamma m \frac{dv_z}{dz} + m v_z \frac{d\gamma}{dz} = -\frac{|e|}{c^2}(\underline{v} \times \underline{B})_z - \nu_\parallel p_z\,, \tag{4}$$

where

$$\frac{d\gamma}{dz} = \frac{-|e|\underline{v}\cdot\underline{E}}{m_o c^3} - \frac{(\nu_\perp p_\perp^2 + \nu_\parallel p_\parallel^2)}{m_o^2 c^3 \gamma}\,.$$

It is straightforward to show that the axial electron acceleration is

$$\frac{dv_z}{dz} = -\frac{c}{2\gamma^2}\frac{\partial a_w^2}{\partial z} + \frac{2a_w a_L k_w c}{\gamma^2}\sin\psi - \frac{2k_w}{k}\nu_\parallel v_z + \frac{3\nu_\perp a_L a_w}{\gamma^2}\cos\psi\,, \tag{5}$$

where $\psi = \theta + \phi = \int_0^z [k + k_w(z') - \omega/v_z(z')]dz' + \psi_0$ is the phase between the electrons and the ponderomotive wave generated by the beating between the radiation and wiggler fields, and ψ_0 is the initial phase at the entrance of the interaction region. Equation (5) can be transformed into the following pendulum equation

$$\frac{d^2\psi}{dz^2} = \frac{dk_w}{dz} - \frac{k}{2\gamma^2}\frac{\partial a_w^2}{\partial z} + \frac{2\, a_w a_L k k_w}{\gamma^2}\sin\psi - \frac{2\nu_\perp k_w}{c} + \frac{3\nu_\perp a_L a_w k}{c\gamma^2}\cos\psi\ . \qquad (6)$$

The rate of change of relativistic energy may be obtained from Eq. (4) and is

$$\frac{d\gamma}{dz} = \frac{a_L a_w k}{\gamma}\sin\psi - \nu_\parallel\gamma + \frac{\nu_\perp a_L a_w}{\gamma}\left(\frac{k}{k_w} - 2\right)\cos\psi - \frac{\nu_\perp}{\gamma}(a_w^2 + a_L^2)\ . \qquad (7)$$

Equations (3), (6) and (7) will be the basic equations we shall use in studying the effects on beam quality due to radiation damping. The terms containing $\cos\psi$ in Eqs. (5), (6) and (7) as well as the last term in Eq. (7) may be neglected when the conditions $a_w^2 \gg a_L^2$, $a_w^2 \gg 1$, $k \gg k_w$, and $\gamma^2 \gg 1$ are satisfied. These conditions are easily achieved in high energy IFEL accelerators.

II. Derivation of Envelope Equation with Radiation Damping

The single particle equations of motion that we have developed in the last section will enable us to study the macroscopic behavior of the beam. This is accomplished by considering the evolution of various averaged quantities associated with the single particle variables.[12,13] We begin by multiplying Eq. (3a) by x' and x, and Eq. (3b) by y' and y, where ' denotes $\partial/\partial z$. Combining the resulting equations yields the following set of equations

$$\frac{1}{2}\frac{d}{dz}\beta_\perp^2 + \frac{K_B^2}{2}\frac{d}{dz}r^2 = -\mu\beta_\perp^2 , \tag{8a}$$

$$\frac{1}{2}\frac{d^2}{dz^2}r^2 - \beta_\perp^2 + K_B^2 r^2 = -\frac{\mu}{2}\frac{d}{dz}r^2 , \tag{8b}$$

$$\frac{d\ell}{dz} = -\mu\ell , \tag{8c}$$

where $r^2 = x^2 + y^2$, $\beta_\perp^2 = x'^2 + y'^2$, $\mu = \gamma'/\gamma + \nu_\perp$, and $\ell = (x'y - y'x)$ is the normalized angular momentum. We eliminated $\beta_\perp^2$ by substituting Eq. (8b) into Eq. (8a). By taking transverse ensemble averages over beam particles in Eq. (8), and denoting the ensemble average of r^2 by $a^2 = \langle r^2\rangle$, we obtain an equation which governs the evolution of the root-mean-square radius of the electron beam,

$$\mu^2\frac{d}{dz}a^2 + \mu\frac{d^2}{dz^2}a^2 + 2\mu K_B^2 a^2 + \frac{d}{dz}\left(\frac{\mu}{2}\frac{d}{dz}a^2\right) + \frac{1}{2}\frac{d^3a^2}{dz^3} + \frac{d}{dz}(K_B^2a^2) + K_B^2\frac{d}{dz}a^2 = 0 . \tag{9}$$

It is easy to show that the integration factor for Eq. (9) is g^2a^2 where $g^2 = \gamma^2\exp(2\int_o^z \nu_\perp dz')$. Equation (9) can now be put into the form $d/dz[\, g^2(a^3a'' + \mu a^3a' + a^4K_B^2)] = 0$, and can be integrated to give $g^2[a^3a'' + \mu a^3a' + a^4K_B^2] = H^2$, where H^2 is a constant of motion associated with the beam. It can be shown that, using the following representation for the particles' normalized transverse velocities,[12]

$$\underline{\beta}_\perp = \frac{a'}{a}r\,\hat{e}_r + \frac{Lr}{a^2}\hat{e}_\theta + \delta\underline{\beta}_\perp ,$$

where $\delta\underline{\beta}_\perp$ is the normalized transverse velocity spread, and $L = \langle\ell\rangle$ from Eq. (8c), the constant H^2 is given by

$$H^2 = \gamma^2(0)L^2(0) + \gamma^2 a^2 \langle|\delta\underline{\beta}_\perp|^2\rangle \exp(2\int_0^z \nu_\perp dz') ,$$

where $\gamma(0) = \gamma(z=0)$ and $L(0) = L(z=0)$. We may therefore define the squared normalized beam emittance[12,16] as $\varepsilon_n^2(z) = \gamma^2 a^2 \langle|\delta\underline{\beta}_\perp|^2\rangle$ and arrive at the following envelope equation

$$\frac{d^2a}{dz^2} + \left(\frac{1}{\gamma}\frac{d\gamma}{dz} + \nu_\perp\right)\frac{da}{dz} + K_B^2 a - \frac{[\varepsilon_n^2(z) + \gamma^2 L^2(z)]}{\gamma^2 a^3} = 0 . \qquad (10)$$

The spatial dependence of the normalized emittance and average angular momentum are given respectively by

$$\varepsilon_n(z) = \varepsilon_n(0)\exp(-\int_0^z \nu_\perp dz') , \qquad (11a)$$

$$L(z) = (\gamma(0)/\gamma)\, L(0)\exp(-\int_0^z \nu_\perp dz') , \qquad (11b)$$

where $\varepsilon_n(0) = \varepsilon_n(z=0)$. Equation (10) together with Eq. (11a,b) constitute the beam envelope equation with radiation damping terms included.

One can see that when $\nu_\perp = 0$, Eq. (11a) shows that ε_n remains constant and Eq. (10) reduces to the usual relativistic beam envelope equation where ε_n is the familiar normalized beam emittance.[12,16] Therefore, in the presence of radiation damping, the root-mean-square beam radius is still described by an envelope equation but the normalized beam emittance is no longer constant but decays exponentially according to Eq. (11a). However, the decay of the normalized beam emittance will eventually be limited by quantum excitation due to the discrete nature of the synchrotron radiation. It is shown in a later section that when an equilibrium is reached between these two competing

processes, the minimum normalized emittance achievable through radiation damping in the IFEL accelerator is given by $(\epsilon_n)_{min} \approx 3\hbar a_w^3 k_w / (\sqrt{2} m_o c K_B)$.

In the presence of radiation damping, the average angular momentum also decays exponentially as given by Eq. (11b). However, one may choose L(0) = 0 for beam generation schemes that do not impart an average angular momentum to the electron beam, i.e., zero magnetic field at the cathode. We shall assume that this is the case in our study of beam quality. We shall also not distinguish between $\nu_\perp$ and $\nu_\parallel$, and will denote both by ν.

III. Evolution of Beam Radius

The equation for the root-mean-square radius a in Eq. (10) is nonlinear. It is found, however, that the mean square radius a^2 satisfies Eq. (9), which is a linear differential equation. For beam focusing provided by the wiggler, Eq. (9) may be solved exactly for untapered wiggler fields when $\gamma' = 0$. If $\gamma' \neq 0$ or when the tapering is known, it can be solved using a WKB method if we assume the coefficients are slowly varying. Equation (9) can be simplified in certain limits of accelerator designs to facilitate analytical study. It can be shown that, $\gamma'/\gamma \ll K_B$ and $\nu \lesssim K_B$, which allow us to arrive at the following approximate equation

$$S''' + 3\mu S'' + 4K_B^2 S' + [4\mu K_B^2 + 2(K_B^2)']S = 0 \,, \qquad (12)$$

where $S = a^2$.

In order to obtain net acceleration of the electrons trapped in the ponderomotive potential, the wiggler field must be spatially tapered. In such a case, the envelope equation, Eq. (12), is a linear differential equation with spatially dependent coefficients. We solved it by using the WKB-method which assumes these coeffients to be slowly-varying functions of longitudinal

distance. By assuming both K_B'/K_B and $\mu \ll K_B$, the general solution to Eq. (12) is found to be

$$S = e^{-M} \frac{K_B(0)}{K_B(z)} [A + B \cos 2\Sigma + C \sin 2\Sigma] ,$$

where $M = \int_0^z \mu(z')dz'$, and $\Sigma = \int_0^z K_B(z')dz'$. The coefficients A, B, C can be found by using the initial conditions for a matched beam, $a(z=0) = a_o$, $a'(z=0) = 0$, $a''(z=0) = 0$. The matched beam radius a_o is related to the initial transverse emittance $a_o^4 = \varepsilon_n^2(0)/(K_B^2(0)\gamma^2(0))$. Using the initial conditions, we arrive at the following expression for the root-mean-square beam radius,

$$a = a_o\, e^{-M/2} \left[\frac{K_B(0)}{K_B(z)}\right]^{1/2} \left[1 + \frac{\mu(0) + K_B'(0)/K_B(0)}{2K_B(0)} \sin 2\Sigma\right]^{1/2} . \qquad (13)$$

Equation (13) shows that the beam radius does not remain constant even when the beam is matched at injection. In addition to the exponential decay from the radiation damping, the beam envelope developes an induced betatron oscillation. However, the normalized emittance is just an exponential decay given by Eq. (11a).

We may gain some insight into the general effect of radiation damping on the transverse emittance by studying Eq. (12) in the case of untapered wiggler field. We shall first consider the case where $\gamma' = 0$. This could be the situation when the acceleration mechanism is saturated by the radiation damping and the beam energy is constant. The evolution of the beam radius is then given by the appropriate limit of Eq. (13). Since there is no tapering of the wiggler, the solution is exact and given by

$$a = a_o e^{-\nu z/2} [\ 1 + \frac{\nu}{2K_B} \sin 2K_B z\ ]^{1/2}\ .$$

The beam radius again exponentially decays with an induced betatron oscillation. Since γ is constant, the damping rate ν is constant, and the normalized emittance ε_n is given by $\varepsilon_n(z) = \varepsilon_n(0) \exp(-\nu z)$.

Next, we consider the situation when an accelerated beam is cooled by passing it through an untapered external wiggler field. Since the beam decelerates due to the synchrotron radiation damping, we have $\gamma'/\gamma = -\nu$. This gives $\mu = 0$ and since $K_B = a_w k_w/(\sqrt{2}\gamma)$, the betatron wave number K_B is a function of z. The spatial dependence of γ can be evaluated using $\gamma'/\gamma = -\nu$, and Eq. (13) reduces to $a \approx a_o(1 + \nu_o^2 z^2)$, where $\nu_o = \tau_R a_w^2 k_w^2 \gamma_o c$. Although the beam radius remains constant up to order of $O(z^2)$, the normalized beam emittance decreases algebraically, $\varepsilon_n = \varepsilon_n(0)/(1+\nu_o z)$.

The relevance of the above analysis depends on the magnitude of the damping rate ν_o. For the following set of accelerator parameters,[2] $E_L = 1.5\times10^9$ V/cm., $B_w = 50$ kG., $\lambda_w = 1$ m, it is estimated that the e-fold length, $1/\nu_o$, could be as short as $\lesssim 600$ m for $\gamma_o = 10^5$. Therefore, our results show that one can improve, by induced synchrotron radiation, the quality of an electron beam by passing it through an external wiggler field.

IV. Quantum Excitation

An estimate for the minimum transverse normalized beam emittance due to quantum excitation in an IFEL accelerator can be obtained from the following qualitative treatment. Similar arguments can be made for electron beams in storage rings.[17,18] The normalized transverse velocity and radial displacement of an electron in a wiggler field are given by $\beta_w = a_w/\gamma$, and $r_w = a_w \lambda_w/(2\pi\gamma)$. For a fluctuation δE in the energy of the electron, the corresponding fluctuations in r_w and β_w are $\delta r_w = \eta\delta E/E$, and $\delta\beta_w = \xi\delta E/E$, where

$\eta=a_w\lambda_w/(2\pi\gamma)$ and $\xi=a_w/\gamma$. The increase in normalized emittance due to such fluctuations is[17,19] $\Delta\varepsilon_n = \gamma[K_B\langle\delta r_w^2\rangle + \langle\delta\beta_w^2\rangle/K_B]$, which for a weakly focusing channel, $K_B \ll k_w$, can be approximated by $\Delta\varepsilon_n \approx \gamma\langle\delta\beta_w^2\rangle/K_B = (\gamma\xi^2/K_B)\langle\delta E^2\rangle/E^2$ Due to the discrete nature of the synchrotron radiation, $\langle\delta E^2\rangle$ is given by $N(\hbar\omega_c)^2$ where $N=Pz/(c\hbar\omega_c)$ is the number of photons emitted in a distance z, P is the synchrotron radiation power, and $\hbar\omega_c$ is the energy associated with a quantum of synchrotron radiation. We can therefore obtain the rate of change of ε_n due to quantum excitation,

$$\left(\frac{d\varepsilon_n}{dz}\right)_{Q.E.} = \frac{\gamma\xi^2}{K_B}\,\frac{P\hbar\omega_c}{cE^2} .$$

However, with radiation damping, the total change in ε_n is given by

$$\left(\frac{d\varepsilon_n}{dz}\right) = -\nu\varepsilon_n + \left(\frac{d\varepsilon_n}{dz}\right)_{Q.E.} .$$

The normalized emittance, ε_n, reaches a minimum, $d\varepsilon_n/dz=0$, when the two effects are balanced. This gives $\varepsilon_n=\gamma\xi^2\hbar\omega_c/(K_B E)$ for the minimum normalized emittance, where we have used $\nu c \approx P/E$. For synchrotron radiation, $\hbar\omega_c=3\hbar c\gamma^3/(2\rho)$ where $\rho=\gamma/(a_w k_w)$ is the radius of curvature of the electron orbit in the wiggler. The minimum transverse normalized beam emittance is then approximately given by

$$(\varepsilon_n)_{min} \approx 3\hbar a_w^3 k_w/(2m_o c K_B) . \tag{14}$$

In the case of weak focusing due to wiggler transverse gradients, $K_B = a_w k_w/(\sqrt{2}\gamma)$, and the minimum normalized emittance is

$$(\varepsilon_n)_{min} \approx 3\hbar\gamma a_w^2/(\sqrt{2}m_o c) . \tag{15}$$

Using the accelerator parameters at the end of section III, Eq. (15) gives the value of the minimum normalized emittance to be ~1.8 cm-rad. Such a large value of the minimum emittance indicates the inadequacy of the

weak focusing from the wiggler transverse gradients. Strong focusing from, for example, a rotating quadrupole field produced by a pair of (or four) helical current windings[20,21] may be required. The betatron wavenumber for such a focusing mechanism[22] is given by $K_B^2 = |e|(\partial B/\partial r)/\gamma m_o c^2$, where $\partial B/\partial r$ is the magnetic field gradient of the quadrupole field on axis. For $\partial B/\partial r \sim 250$ G/cm, $a_w=600$, $\lambda_w=10$m, and $\gamma=4\times10^5$, Eq.(14) gives a minimum normalized emittance of $\varepsilon_n \sim 0.13$ cm-rad. Another possible strong focusing force could be the electrostatic radial electric field of an ion column. Such a column could be created by the ionization of the residual gas by a low energy, high current electron beam pulse preceeding the main accelerating beam pulse.[23-25] The betatron wavenumber for such a focusing mechanism can be easily shown to be $K_B^2 = \omega_{pi}^2(m_i/m_o)/(2\gamma c^2)$, where ω_{pi} is the ion plasma frequency and (m_i/m_o) is the mass ratio between the ions and the electrons. For $n_i=10^{12}/\text{cm}^3$, $a_w=600$, $\lambda_w=10$m, and $\gamma=4\times10^5$, Eq. (14) gives a minimum normalized emittance of $\varepsilon_n \sim 0.04$ cm-rad. An additional benefit of having ion focusing in the IFEL accelerator is that the radial plasma electron density profile in an ion column can also be a focusing medium for the laser beam.

V. Numerical Example

We shall consider only resonant particles whose phase ψ satisfies the conditions $d\psi/dz = 0$ and $d^2\psi/dz^2 = 0$. The first condition gives

$$\gamma_R = \frac{|e|\sqrt{k}}{\sqrt{2}m_o c^2} B_w k_w^{-3/2}, \tag{16a}$$

$$\gamma_R' = R_1 k_w^{1/2} - R_2 B_w^4 k_w^{-3}, \tag{16b}$$

$$\nu = \frac{\sqrt{2}}{3} \frac{|e|^5}{m_o^4 c^8} B_w^3 k_w^{-3/2} \sqrt{k} , \tag{16c}$$

where $R_1 = \sqrt{2}|e|E_L \sin\psi_R/(m_o c^2\sqrt{k})$, $R_2 = |e|^6 k/(3m_o^5 c^{10})$, ψ_R is the resonance phase, E_L the laser electric field strength, and k the laser wave number. The second condition together with the pendulum equation, Eq. (6), provide the spatial dependences of k_w and B_w,

$$3\, k_w' - \frac{2k_w}{B_w} B_w' + \frac{4E_L}{k} \sin\psi_R \frac{k_w^3}{B_w} - \frac{2\sqrt{2}mc^2}{|e|\sqrt{k}} R_2 \frac{B_w^3}{\sqrt{k_w}} = 0. \qquad (17)$$

Equation (17) shows that the required tapering of the wiggler field may be obtained by prescribing ψ_R and a relationship between k_w and B_w in Eq. (17). As an example, we assume the tapering of the wiggler field to be that of a maximum rate IFEL accelerator.[2] For such a case the wiggler strength and the wiggler period are related by the following power law,

$$B_w = (R_1/6R_2)^{1/4}\, k_w^{7/8} .$$

Equation (17) may then be solved to give

$$B_w = B_w(0)[1 + R_4 z]^{-7/9}, \qquad (18a)$$

$$k_w = k_w(0)[1 + R_4 z]^{-8/9}, \qquad (18b)$$

where

$$R_4 = \frac{9\sqrt{2}mc^2}{|e|\sqrt{k}} R_2(R_1/6R_2)^{3/7} B_w(0)^{9/7} .$$

Evaluating Eq. (11) and (16a) with (16c) and (18a,b) gives the normalized transverse emittance and the resonant energy of the beam as functions of the propagation distances.

For our example, we will consider the following set of accelerator parameters[2]: $E_L = 1.5\times10^9$ V/cm, $B_w(0) = 50$ kG, $\lambda_w(0) = 100$ cm, $\lambda = 10.6$ μm and a resonance phase of $\sin\psi_R = 0.6$. The initial conditions are for a matched beam with a radius of 1 mm , a normalized emittance of $\varepsilon_0 \approx .205$ cm-rad, and the required beam injection energy is ~ 52 GeV. The beam is allowed to propagate for 1 km without depleting the laser radiation. We repeated the calculation by assuming there is no radiation damping but with the same power law tapering of the wiggler field.

The results are represented in Figs. (1), (2), and (3). The open squares denote the presence of radiation damping, while open circles denote its absence. From Fig. (1), we can see that the final energy is not significantly reduced by the radiation damping. Figure (2) shows the exponential decay of the normalized emittance. At the end of the one-kilometer accelerator, the normalized emittance is reduced to .0[illegible] cm-rad which is very close to the minimum normalized emittance of ~0.0[illegible] cm-rad at that point if ion column focusing is assumed in the accelerator. In Fig. (3), the appropriate tapering of k_w and B_w for the two cases are shown.

Conclusion

We have studied the evolution of transverse emittance and the beam radius due to the radiation damping effect in an IFEL accelerator. We derived the beam envelope equation, Eq. (10), which includes the effects of radiation damping, and have demonstrated that the normalized transverse emittance decreases exponentially with a damping rate given by the radiation damping coefficient ν until it reaches a minimum value due to quantum excitation The beam envelope equation was solved analytically for a slowly-varying wiggler field. We have derived an expression for the minimum normalized emittance in the IFEL accelerator and showed that strong focusing is essential

in reducing this minimum emittance due to quantum excitation. We have shown that radiation damping can play an important role in improving beam quality without a significant sacrifice in beam energy.

Acknowledgement

This work was supported by the U.S. Department of Energy under Contract No. DE-AI05-83-ER40117.

The authors appreciate discussions with C. M. Tang, D. Sutter, T. Godlove, L. Blumberg, and P. Morton.

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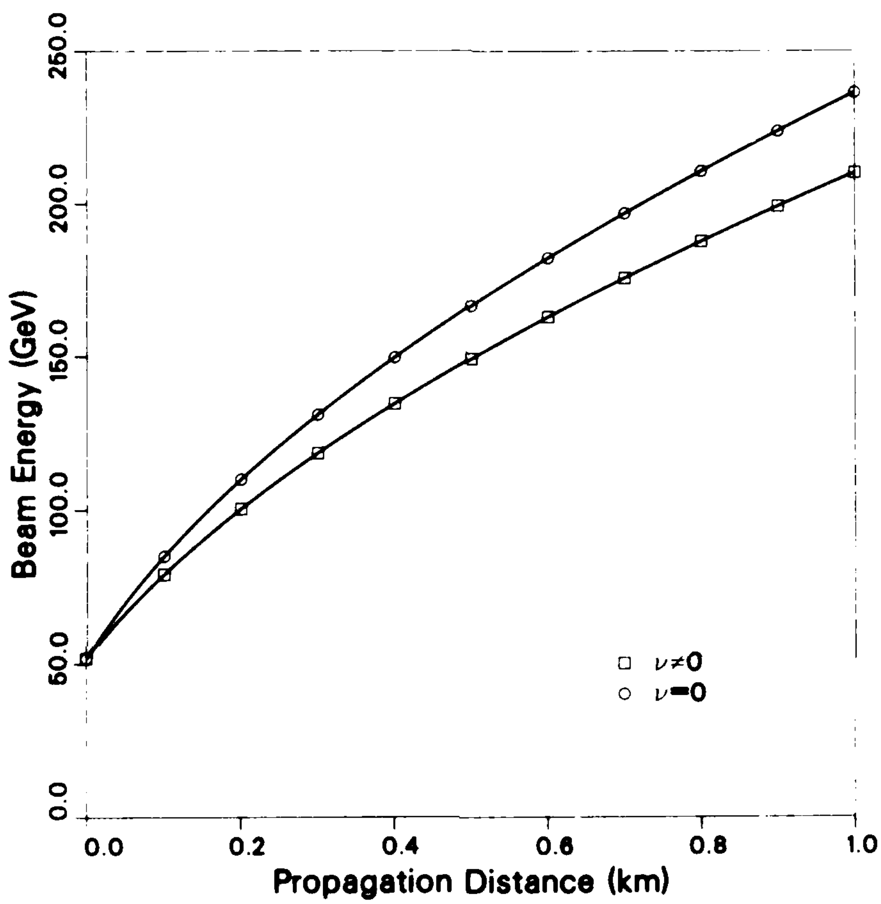

Fig. 1 Evolution of beam energy in an IFEL accelerator with and without radiation damping.

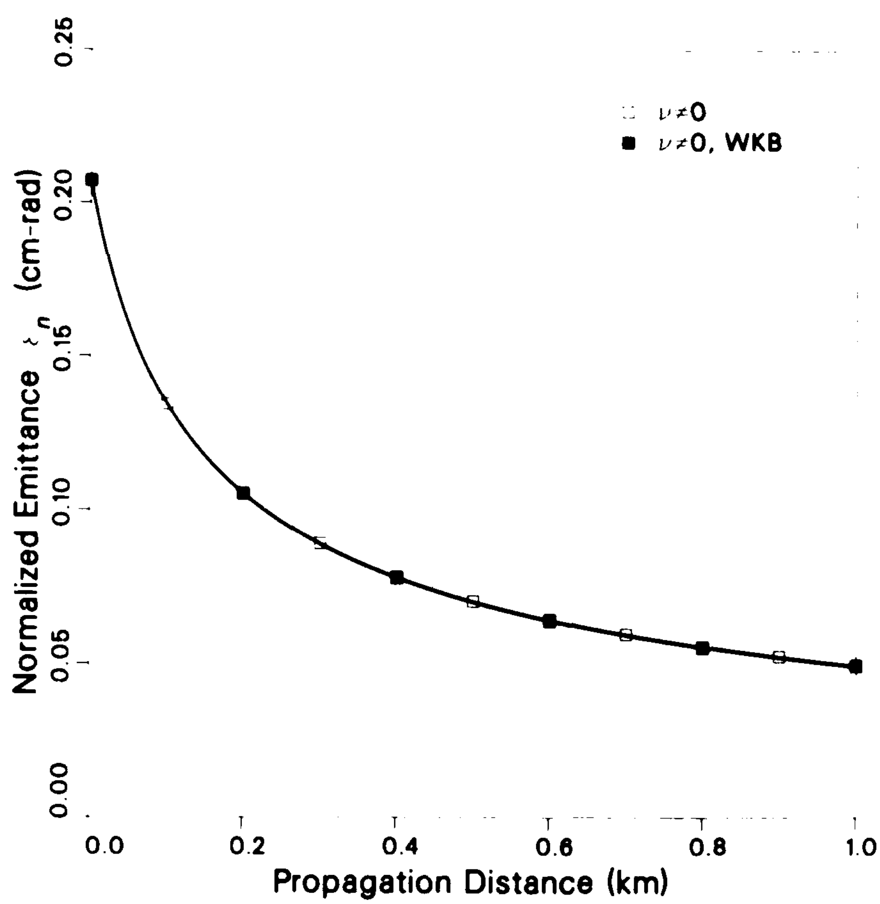

Fig. 2   Exponential decay of normalized beam emittance, ϵ_n^2

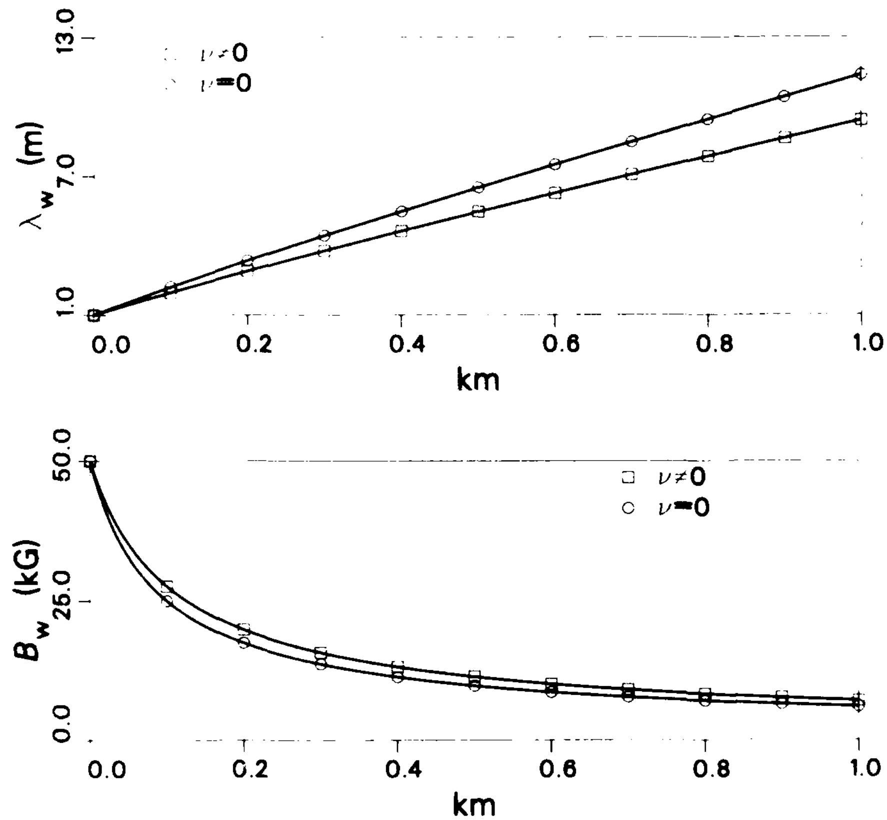

Fig. 3 Spatial tapering of wiggler period and field with and without radiation damping.

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